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UNITED STATES NAVAL POSTGRADUATE SCHOOL

A NONVARYING-C* CONTROL

SCHEME FOR AIRCRAFT

by

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ABSTRACT:

A sum of normal acceleration and pitch rate appears to be the best variable to use to control aircraft in the longitudinal axis. The C*-Criterion specifies that the time response of this quantity called C* must fall in a prescribed envelope for all speeds and altitudes. It is equivalent to requiring the control system to hold the coefficients of a certain equation, which describes the aircraft's short-period motion, fixed. This is done by using feedbacks with variable gains. The gain-changing mechanism is found using gradient techniques.

The system was shown to be practical by an analog simulation. It was found to be tolerant of instrument noise, elevator hysteresis, and other complications not accounted in the analytical derivations.

The study strongly suggests a modification of the C*-Criterion. It is proposed that the coefficients in the expression of the quantity C* should be representative of the aircraft's parameters rather than have the values currently in use.

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FOREWORD

Three Master's theses, currently in progress, are extensions of the studies outlined in this report and will use it as reference. The theses are:

"An Investigation of Oscillations in an Adaptive Aircraft Control System under Large Input Commands," by Ens. L. S. Wisler, USN.

"A Study of an Adaptive Aircraft Control System in a Self-Organizing Configuration," by Lt. H. M. Richarde, USN.

"The Extension of an Adaptive Aircraft Control Concept to Helicopters," by Lt. J. M. Hood, USN.

Most of the work reported was done while the author was employed by Honeywell, Inc. The analytical derivations are presented but only the conclusions derived from supporting studies with an analog computer are given.

LIST OF ILLUSTRATIONS

Adaptive Control for Aircraft

The usual method for accommodating an automatic flight control system to the wide variations in dynamic characteristics of the airframe, with changes of airspeed and altitude over its flight envelope, is to change control-loop gains with measurements of air-data. The dynamic pressure or the Mach number, as estimated by an air-data computer, is generally used as the parameter with which the gains are scheduled. The electronic art, developing at a bewildering pace, is used to provide reliable, high-performance control hardware. All flight control systems currently in production, with the exception of an adaptive system for the F-111 and some fixed-gain controllers, use air-data scheduling. This includes the system for the giant C-5A and flight controls proposed for the new cycle of commercial transport aircraft. However, the engineering analysis required for design is extensive and the final adjustments to tailor the control system to the aircraft during flight tests are expensive, so there has been the hope that methods of control would be found which did not rely on scheduling with air-data but changed parameters by measurements of dynamic performance. These would ultimately lead to much simpler, more reliable and more universally applicable systems.

A large number of such schemes has been proposed in the last fifteen years. Some have been studied in flight tests and only one has gone into production. A brief survey of the older ideas is given by Blakelock, Reference 1. Boskovich and Kaufmann, Reference 2, describe the evolution of a successful high-gain, wide bandwidth, model-follower system. The Bendix system, described by J. Bernadyn, et al., Reference 3, was flown. The North American

SIDAC Controller, developed by Shipley and his associates, References 4 and 5, has been extensively analyzed and completely designed but has been held short of flight test because of lack of funds. These devices, while being very successful in most respects, have debilitating characteristics.

Controllers which detect limit cycles, either intentionally, as in the Honeywell device for the X-15, or unintentionally, as in the General Electric control system for the F-111, are upset by air turbulence unless special precautions are taken which then reduce the effectiveness and increase the complexity of the system. We conjecture that the Bendix OLAC was not carried further because it probably did not control adequately at all flight conditions. The SIDAC will probably show unwanted oscillations under large commands. It is a high-gain model-follower and is complicated. Since reliability, ease of maintenance, effect of component failures and results of battle damage are vital considerations besides performance in control system evaluation, the relatively complex adaptive schemes have been losing favor. Indeed, some people, for example Horowitz in Reference 6, argue that linear control should be adequate.

A somewhat different point of view has been taken by R. C. Hendrick in developing a system, described in Reference 7, currently undergoing flight testing at Patuxent River. His approach is to use only the minimum amount of information from the dynamical motion that is needed for the single control system to be applicable to a whole class of aircraft without additional tailoring.

Our study reverts to the goal of providing a response that is uniform for all flight conditions without using measurements of dynamic pressure, altitude

or angle-of-attack. Information obtained from inertial instruments, such as accelerometers or rate gyros, or from measurements of control surface positions and command inputs is admissible. We persist in this area despite the general disenchantment with adaptive controls not only because it appears that a really practical solution for aircraft is close at hand but also because the study of the problem yields basic understanding of aircraft controls and the difficult question of requirements for suitable handling qualities.

The Approach

Our scheme is very close to that used for SIDAC, (4, 5). The modification is based on the observation that the equation for handling-qualities C*-Criterion is very similar to a basic short-period equation for the motion of the aircraft. A modest feedback and feedforward with variable gains holds the coefficients of the equation fixed. The C*-requirement may be met by choosing these coefficients to be the same as demanded by the Criterion or by adding a fixed outer loop. The mechanism for varying the gains is found by a gradient calculation similar to the SIDAC-analysis. We achieve the advantage of a low-gain, narrow-bandwidth system which is very insensitive to instrument noise, bending modes, accommodates the primary control system and satisfies the requirements directly rather than using a model-following technique.

The SIDAC-System identifies parameters and uses this in turn to adjust gains. Since the accuracy of identifying the several coefficients depends on the frequency content of the motion and the particular flight condition, it appears that a system which calls for the required response directly should have an advantage. This has also been argued by Hofmann and Best, Reference

8, in a fairly similar approach to control of the lateral-directional axes.

The methods of synthesizing the gain changing mechanism have been developed by many authors. We studied a Lyapumov-function technique, reported in Reference 9. This was later rediscovered by Parks, Reference 10. P. M. Lion, Reference 11, has generalized and unified the several methods.

The C*-Criterion

The most difficult problem in flight control design, besides considerations of making the system invulnerable to component failures, is in deciding on what flying characteristics will be acceptable to the pilot. The requirement for longitudinal response which best fits into an analytical formulation is that promulgated by Tobie, Elliot and Malcom, Reference 12. The criterion is that the time-response trace of a quantity called C*, for an abrupt force applied to the stick, must fall within a certain envelope, shown in Figure 1. C* is the sum of the normal force applied to the pilot's seat plus a constant multiple of the angular velocity in pitch. Thus, it is a linear combination, with constant positive coefficients, of normal acceleration at the aircraft's center-of-gravity, the pitch velocity and the pitch acceleration, assuming the pilot's station is ahead of the cg. This blended response variable has been used for flight controls before. It is a reasonable quantity to consider because at flight conditions with high dynamic pressures, at which the aircraft is very responsive, the pilot likes to feel the normal acceleration with stick force. He bases his assessment of the aircraft's performance on it. However, in low-q flight conditions, for example in landing, pitch rate is a more important clue to the aircraft's controllability. Indeed, for a helicopter at hover, pitch attitude is the variable

which must be controlled. The sum of the quantities exhibits this change in emphasis. The steady-state response of normal acceleration has the factor of the aircraft's velocity when compared to that for the pitch rate and hence normal acceleration will dominate at high speeds and pitch rate will be important at low speeds. A more comprehensive analysis is given in the report by Makers, Reference 13.

A Conjecture

The study of our system by analog simulation showed that it was necessary to hold the coefficients of the short-period equation fixed as values representative of the ranges of the aircraft's coefficients over the flight envelope and not to achieve the C*-values directly. Analysis presented later also shows this. It is necessary because of higher order effects that are ignored in the basic derivation but are present in realistic situations. The C*-criterion is met by using an outer-loop feedback with fixed coefficients.

This naturally prompts a review of the C*-requirement. For the F-4 aircraft, which was used as the vehicle for the study, the pilot sits only twelve feet ahead of the center-of-gravity. This makes the effect of pitch acceleration on the C*-quantity very weak. Since most of the studies of C* were done on fixed-base simulators, it appears possible that the angular-acceleration clue is not properly accounted in the criterion. Some studies, for example, done by Bihrle, Reference 14, indicate that it is an important clue. Thus, we conjecture that a more natural criterion for longitudinal handling-qualities is that the coefficients of the C*-expression be taken in the range of the coefficients of the corresponding short-period equation.

Not only does this make our adaptive scheme more natural but it appears that the envelope of allowable responses, Figure 1, may be made much more restrictive. Then the Criterion becomes more definitive and easier to apply. Further, it can be directly extended to include helicopters by adding a term for pitch attitude. The corresponding short-period equation for the helicopter in hover has just such a term and this quantity is an important handling cue in this condition. Thus, this modification of C* is intuitively appealing.

Equations for Short-Period Motion

We begin with the equations of perturbations from straight and level flight, written as

$$\left\{\begin{aligned} \ddot{\theta} &= M_q \dot{\theta} + M_\alpha \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_\delta \delta \\ n &= U_0 (\dot{\theta} - \dot{\alpha}) = -Z_\alpha \alpha - Z_\delta \delta , \end{aligned}\right. \tag{1}$$

in terms of angular rate in pitch $\dot{\theta}$, angle-of-attack α, and normal acceleration at the center-of-gravity n. The quantity U_0 is the value of the aircraft's unperturbed velocity and δ represents the elevator deflection. The coefficients M_δ, Z_α and Z_δ are constants, representative of the flight condition, and have appropriate dimensions. All angles are measured in radians and linear quantities are measured in feet. Data for the F-4 aircraft is given in Table 1.

Angle-of-attack is difficult to measure, and its interpretation is complicated by turbulence in the air, so, following Shipley (4), we eliminate it from the equations and find

$$\left\{\begin{aligned} \ddot{\theta} &= \bar{\beta}_q \dot{\theta} + \bar{\beta}_n n + \bar{\beta}_\delta \delta \\ \dot{n} &= -z_\alpha \dot{\theta} + \frac{z_\alpha}{U_0} n - z_\delta \dot{\delta} \end{aligned}\right. \tag{2}$$

The coefficients are

$$\left\{\begin{aligned} \bar{\beta}_q &= M_{\dot{\alpha}} + M_q \\ \bar{\beta}_n &= -M_\alpha / z_\alpha - M_{\dot{\alpha}} / U_0 \\ \bar{\beta}_\delta &= M_\delta - z_\delta M_\alpha / z_\alpha \end{aligned}\right. \tag{3}$$

and are listed in Table 1. They are negative for all flight conditions. The first equation in Set 2 is the fundamental relation in this study.

The C*-quantity is

$$C^* = n + \ell \ddot{\theta} + U_c \dot{\theta} \tag{4}$$

in which ℓ is the distance of the pilot forward of the cg and U_c is a number called the cross-over velocity, usually taken around four hundred feet per second. If we require C* to be precisely a multiple of the command input C_c ,

$$c^* = -k\,C_c\,, \tag{5}$$

Equation 4 may be written as

$$\ddot{\theta} = -\frac{u_c}{\ell}\,\dot{\theta} - \frac{1}{\ell}\,n - \frac{k}{\ell}\,C_c\,. \tag{6}$$

This is exactly the form of the fundamental equation in (2).

The Basic Derivation

The control configuration is diagrammed in Figure 2. To avoid complicating the discussion, for the present neglect the actuator, actuator model, the fixed feedback and the dynamics of the primary control system. Thus,assume for now that

$$\begin{cases} \delta = c + f \\ \bar{c} = c = C_c\,. \end{cases} \tag{7}$$

We are looking for a control function

$$f = \Gamma_q\,\dot{\theta} + \Gamma_n\,n + \Gamma_\delta\,c \tag{8}$$

in which the gains Γ_q, Γ_n, Γ_δ vary so that for each flight condition

the over-all system behaves as if the aircraft equation

$$\ddot{\theta} - \bar{\beta}_q \dot{\theta} - \bar{\beta}_n n - \bar{\beta}_\delta \delta = 0 \tag{9}$$

were replaced by one with given fixed coefficients β_q, β_n, β_δ:

$$\ddot{\theta} - \beta_q \dot{\theta} - \beta_n n - \beta_\delta c = 0. \tag{10}$$

Hence, we define an error signal, on which the generation of the variable gains is based, by the amount with which Equation 10 fails, namely

$$\mathcal{E} = \ddot{\theta} - \beta_q \dot{\theta} - \beta_n n - \beta_\delta c. \tag{11}$$

Subtracting (9) from (11) and incorporating the control function yields

$$\begin{aligned} \mathcal{E} = \left[\bar{\beta}_q - \beta_q + \bar{\beta}_\delta \Gamma_q\right]\dot{\theta} + \left[\bar{\beta}_n - \beta_n + \bar{\beta}_\delta \Gamma_n\right] n \\ + \left[\bar{\beta}_\delta - \beta_\delta + \bar{\beta}_\delta \Gamma_\delta\right] c. \end{aligned} \tag{12}$$

Therefore, to hold the error zero the gains must be adjusted to the values

$$\Gamma_q^* = \frac{\beta_q - \bar{\beta}_q}{\bar{\beta}_\delta}, \quad \Gamma_n^* = \frac{\beta_n - \bar{\beta}_n}{\bar{\beta}_\delta}, \quad \Gamma_\delta^* = \frac{\beta_\delta - \bar{\beta}_\delta}{\bar{\beta}_\delta}. \tag{13}$$

Values of these with

$$\beta_q = -3 \quad , \quad \beta_n = -.02 \quad , \quad \beta_s = -20$$

are listed in Table 1.

To construct the mechanism for calculating the gain, we follow Shipley in considering the gradient of the sum of squares of the coefficients of Equation (12). This method is now a standard technique. Let

$$2V = \frac{1}{K_q} B_q^2 + \frac{1}{K_n} B_n^2 + \frac{1}{K_s} B_s^2 \tag{14}$$

where

$$\left\{ \begin{aligned} B_q &= \bar{\beta}_q - \beta_q + \bar{\beta}_s \Gamma_q \\ B_n &= \bar{\beta}_n - \beta_n + \bar{\beta}_s \Gamma_n \\ B_s &= \bar{\beta}_s - \beta_s + \bar{\beta}_s \Gamma_s \end{aligned} \right. \tag{15}$$

and K_q, K_n, K_s are positive numbers. Then differentiation of (14) with times gives

$$\frac{dV}{dt} = \bar{\beta}_s \left[\frac{1}{K_q} B_q \frac{d\Gamma_q}{dt} + \frac{1}{K_n} B_n \frac{d\Gamma_n}{dt} + \frac{1}{K_s} B_s \frac{d\Gamma_s}{dt} \right]. \tag{16}$$

Hence, choosing

$$\left\{ \begin{aligned} \frac{d\Gamma_q}{dt} &= K_q \dot{\theta} G \\ \frac{d\Gamma_n}{dt} &= K_n n G \\ \frac{d\Gamma_s}{dt} &= K_s c G \end{aligned} \right. \tag{17}$$

results in the equation

$$\frac{dV}{dt} = \bar{\beta}_s \varepsilon G. \tag{18}$$

Since $\bar{\beta}_s < 0$, choosing G as

$$G = \varepsilon \tag{19}$$

or

$$G = \operatorname{sgn} \varepsilon = \begin{cases} +1, & \varepsilon > \varepsilon_0 \geq 0 \\ 0, & -\varepsilon_0 \leq \varepsilon \leq \varepsilon_0 \\ -1, & \varepsilon < -\varepsilon_0 \end{cases} \tag{20}$$

insures that $\frac{dV}{dt}$ is negative. Hence, we expect that V will tend to zero and the gains will tend to the ideal values at each flight condition. The system is not stable in any classical sense because the manner in which the gains converge depends on the activity and frequency content of the input. However, a concept of eventual stability has been used by LaSalle and Rath, Reference 15, to describe the mathematical behavior of such systems.

We prefer the mechanization given by (20). Shipley at first used (19) and found that an automatic gain-control circuit was needed to provide stability under large inputs. The final SIDAC system (5) uses not only the sign of the error but also the sign of the dependent variables to form the quantities which when integrated produce the varying gains. In our system the gains are found from the equations

$$\left\{\begin{aligned} \Gamma_q &= K_q \int \dot{\theta}\ \mathrm{Sgn}\,\varepsilon\, dt \\ \Gamma_n &= K_n \int n\ \mathrm{Sgn}\,\varepsilon\, dt \\ \Gamma_\delta &= K_\delta \int c\ \mathrm{Sgn}\,\varepsilon\, dt \end{aligned}\right. \tag{21}$$

with $K_q = 5 \quad , \quad K_n = 5 \times 10^{-5}, \quad K_\delta = 500, \quad \varepsilon_0 = .02.$

Modifications to Include the
Dynamics of the Actuator

Returning to Figure 2 we now include the actuator, considered as a first order lag, add the corresponding model of the actuator, and the fixed-feedback outer-loop. The equations are then modified to read

$$\left\{\begin{aligned} \ddot{\theta} &= \bar{\beta}_q \dot{\theta} + \bar{\beta}_n n + \bar{\beta}_\delta \frac{c+f}{T_a s+1} \\ \varepsilon &= \ddot{\theta} - \beta_q \dot{\theta} - \beta_n n - \beta_\delta \frac{c}{T_a s+1} \\ c &= c_c + H_q \ddot{\theta} + H_q \dot{\theta} + H_n n . \end{aligned}\right. \tag{22}$$

Since the Laplace operator is equivalent to differentiation, the first two equations may be rewritten as

$$\left\{\begin{aligned} (T_a s+1)\varepsilon &= (T_a s+1)\ddot{\theta} - T_a \beta_q \ddot{\theta} - T_a \beta_n \dot{n} - \beta_n n - \beta_\delta c \\ (T_a s+1)\ddot{\theta} &= T_a \bar{\beta}_q \ddot{\theta} + \bar{\beta}_q \dot{\theta} + T_a \bar{\beta}_n n + \bar{\beta}_n n \\ &\quad + \bar{\beta}_\delta (c+f). \end{aligned}\right. \tag{23}$$

Combining these two and including the evaluation of the control function (8) gives

$$(T_a s+1)\,\varepsilon = T_a(\bar{\beta}_q - \beta_q)\ddot{\theta} + T_a(\bar{\beta}_n - \beta_n)\dot{n} + (\bar{\beta}_q - \beta_q + \bar{\beta}_\delta \Gamma_q)\dot{\theta} + (\bar{\beta}_n - \beta_n + \bar{\beta}_\delta \Gamma_n)n + (\bar{\beta}_\delta - \beta_\delta + \bar{\beta}_\delta \Gamma_\delta)C. \tag{24}$$

Thus, the actuator introduces the first two terms on the right hand side which were not found in Equation 12. These could be taken into account by adding two more feedbacks of $\ddot{\theta}$ and n with varying gains. We avoid this complication and hope that these terms will not lead to serious errors. Analog simulation shows that this is the case.

The requirement given by Equation 5, that C* should be a multiple of the command input is unrealistic, considering the vast latitude by Figure 1. We modify this by requiring that C* follow the command by a simple lag,

$$C^* = \frac{-k}{T_c s + 1} C_c, \tag{25}$$

and calculate the deviation from this by

$$\varepsilon_c = C^* + \frac{k}{T_c s + 1} C_c. \tag{26}$$

This is the equivalent to

$$(T_c s+1)\,\varepsilon_c = T_c \dot{n} + n + \ell T_c \dddot{\theta} + (\ell + U_c T_c)\ddot{\theta} + U_c \dot{\theta} + k C_c \tag{27}$$

when the definition of C*, Equation 4, is used. The first equation of (23), upon adding the fixed feedback, becomes

$$
\begin{aligned}
(T_a s+1)\varepsilon = {} & T_a \dddot{\theta} - T_a \beta_n \dot{n} + (1 - T_a \beta_q - \beta_s H_{\dot{q}})\ddot{\theta} \\
& - (\beta_q + \beta_s H_q)\dot{\theta} - (\beta_n + \beta_s H_n) n - \beta_s C_c .
\end{aligned}
\tag{28}
$$

Multiplying Equation 28 by a constant and subtracting it from Equation 27 yields

$$
\begin{aligned}
& (T_c s+1)\varepsilon_c - \mu (T_a s+1)\varepsilon = (T_c l - \mu T_a)\dddot{\theta} \\
& + (T_c + \mu T_a \beta_n)\dot{n} + (l + T_c U_c - \mu + \mu T_a \beta_q + \mu H_{\dot{q}})\ddot{\theta} \\
& + (U_c + \mu \beta_q + \mu \beta_s H_q)\dot{\theta} + (1 + \mu \beta_n + \mu \beta_s H_n) n \\
& + (k + \mu \beta_s) C_c .
\end{aligned}
\tag{29}
$$

We now have the quantities $H_{\dot{q}}$, H_q , H_n , μ and k at our disposal to make the C*-error small. It is assumed that the gain changers hold the performance error ε small. Thus, require

$$
\left\{
\begin{aligned}
& k + \mu \beta_s = 0 \\
& 1 + \mu \beta_n + \mu \beta_s H_n = 0 \\
& U_c + \mu \beta_q + \mu \beta_s H_q = 0 \\
& l + T_c U_c - \mu + \mu T_a \beta_q + \mu \beta_s H_{\dot{q}} = 0 .
\end{aligned}
\right.
\tag{30}
$$

The first equation gives us a gain relation, the others define the fixed-feedback gains as

$$
\left\{
\begin{aligned}
H_n &= - \frac{1 + \mu \beta_n}{\mu \beta_s} \\
H_q &= - \frac{U_c + \mu \beta_q}{\mu \beta_s} \\
H_{\dot{q}} &= - \frac{l + T_c U_c - \mu + \mu T_a \beta_q}{\mu \beta_s}
\end{aligned}
\right. .
\tag{31}
$$

The quantity μ can now be taken as a gain for a root locus study or to kill off one of the two remaining terms in Equation (29). With the data

$$\ell = 12 \qquad T_a = 0.1 \qquad \beta_\delta = -20,$$
$$T_c = .25 \qquad \beta_q = -3$$
$$U_c = 300 \qquad \beta_n = -.02$$

the variations of these gains, the open-loop steady-state gain G_o, and the resulting closed-loop roots are shown in Table 2, (the ideal feedbacks (13) are assumed). We note that the open-loop gain is not very large. Bode plots show that the bandwidth is always less than 12 radians per second. This analysis was made from the transfer functions recorded in the Appendix.

Performance of the System

The controller was evaluated on an all-analog simulation. After initial study with only the actuator as the complication, a representation of the primary control system, a second-order, forty-radian-per-second servo, a simulation of 0.1-degree elevator hysteresis, and equations describing a bending mode were added. The system accepted these although the rates and accuracies for the convergence of the gains were reduced. The dynamics of the second-order servo introduced limit cycles under relatively large command inputs. The effects of noisy measurements and air turbulence were studied. The system was not upset by these. The most notable aspect of the performance was the insensitivity of the C*-response to large deviations of the varying gains from their ideal values.

The fixed outer feedback could be eliminated by choosing the parameters of the error expression (11), β_q , β_n , β_δ to be equal to the corresponding

values in the C*-equation (16). The simulation showed that with this choice the gain-changing mechanism was unstable. To keep the loop-gains and the errors introduced by neglected complications small, these parameters are picked to be in the range of the corresponding aircraft parameters $\bar{\beta}_q$, $\bar{\beta}_n$, $\bar{\beta}_\delta$. Then the system is stable.

Conclusion

The Nonvarying-C* Control Scheme has been shown to have practical application for fixed-wing aircraft. It should be possible to extend the method for use on helicopters. The approach of choosing the variable-gain mechanism on the basis of system performance rather than requiring an explicit parameter identification results in a low-bandwidth system that is very tolerant to noise and high-order effects. The study also suggests a modification of the C*-Criterion for longitudinal handling qualities.

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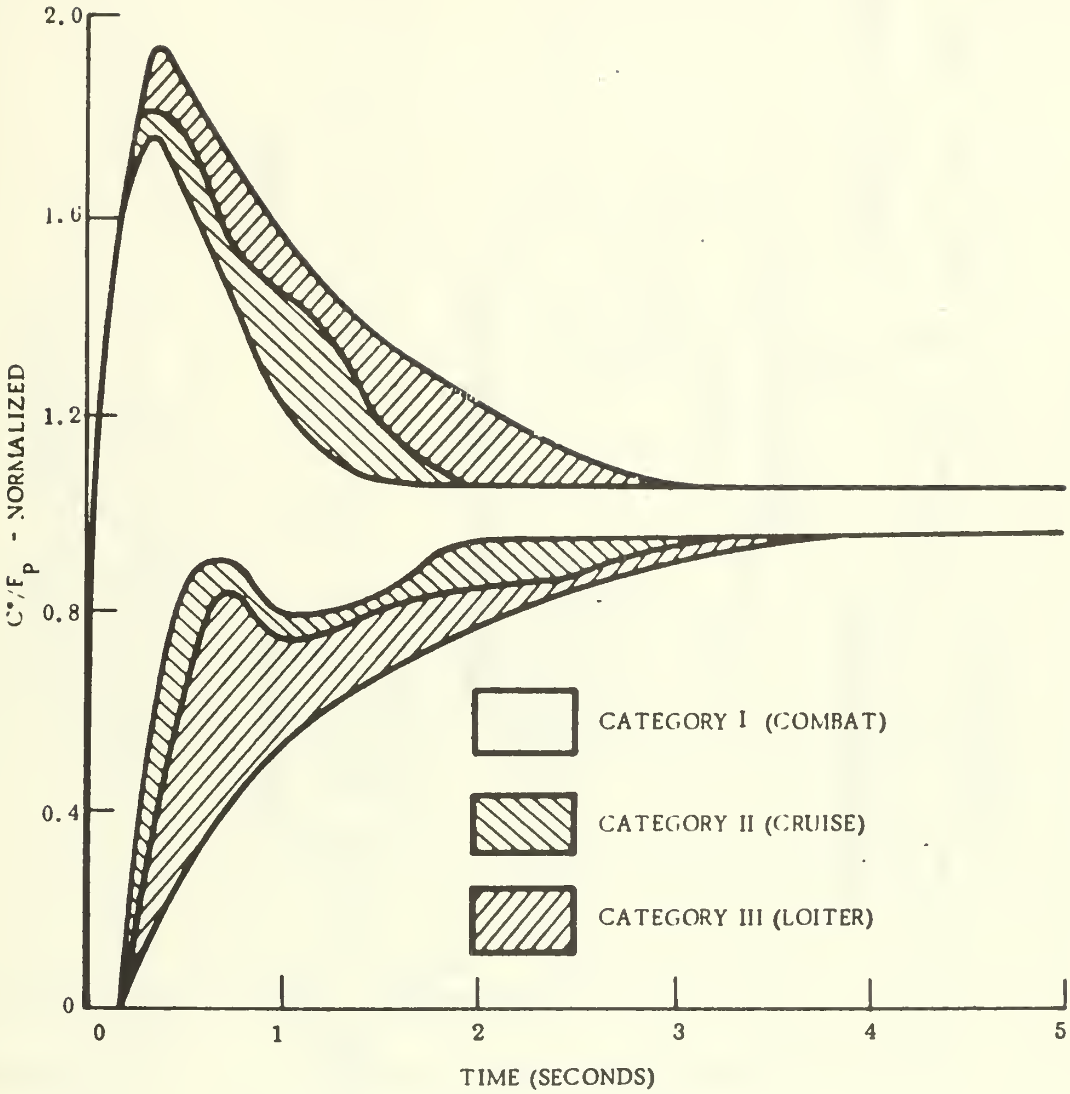

FIGURE 1. C*-STEP-RESPONSE ENVELOPES

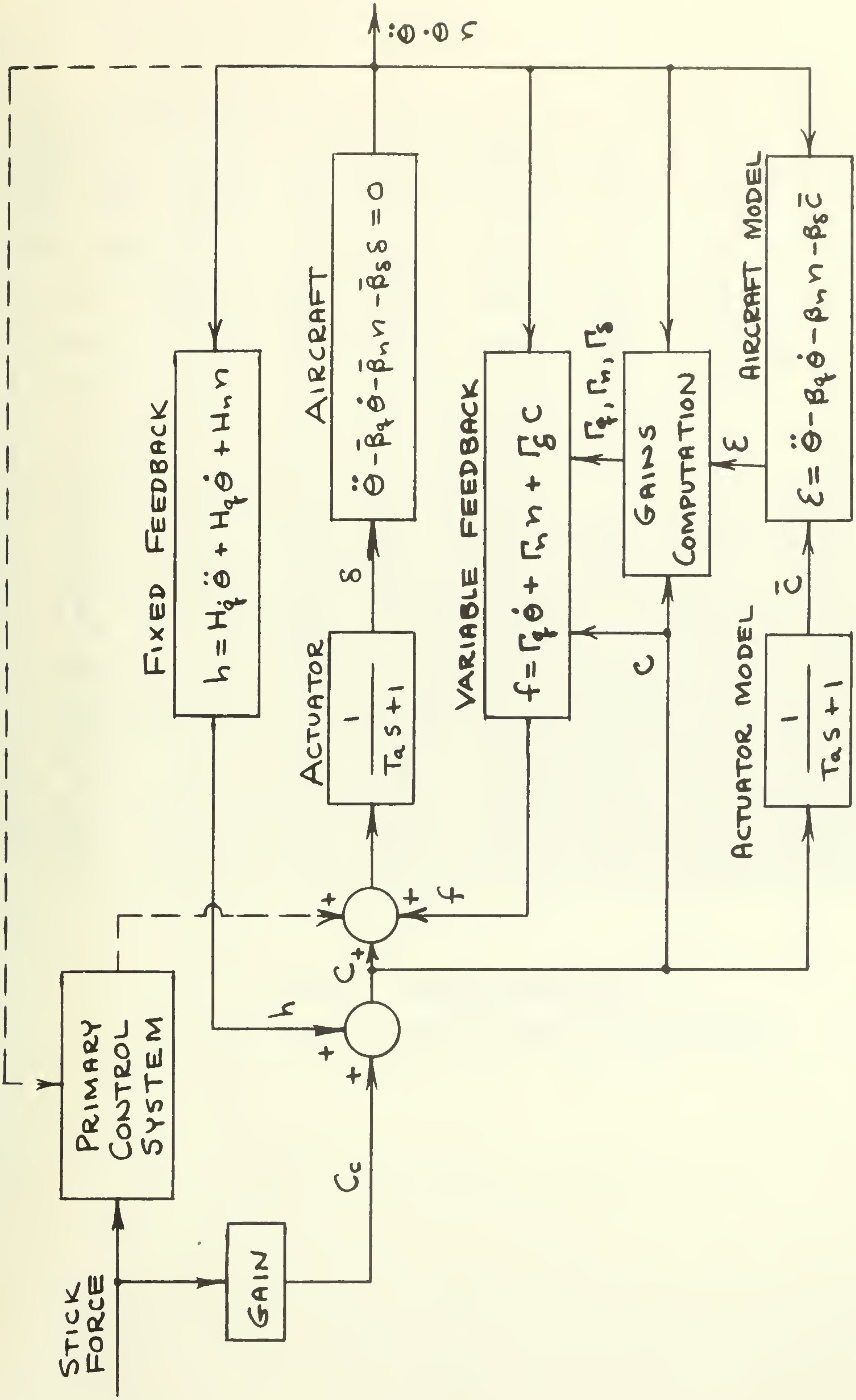

FIGURE 2. THE NONVARYING-C* CONFIGURATION

FLIGHT COND.	1	2	3	4	5	6	7	8
Alt. $\times 10^{-3}$	SL	15	30	SL	SL	15	30	50
Mach No.	0.2	0.4	0.6	0.9	1.2	1.5	2.0	2.0
U_0	223	423	597	1005	1340	1587	1990	1938
$-M_\alpha$	1.3	2.93	3.6	38	180	157	110	42.4
$-M_{\dot\alpha}$	.26	.33	.27	1.38	1.33	.40	.10	.04
$-M_q$	.43	.56	.51	2.61	3.84	2.40	1.21	.48
$-M_\delta$	2.8	6.3	7.5	58.4	101	65.0	38.6	14.9
$-Z_\alpha$	83	228	262	2420	4421	3162	2119	817
$-Z_\delta$	13.2	31.5	38.2	313	600	401	265	102
$-\bar\beta_\delta$	2.58	5.90	6.93	53.5	76.6	45.1	24.8	9.61
$-\bar\beta_q$	.69	.89	.78	4.0	5.2	2.8	1.3	.52
$-\bar\beta_n$	.0146	.0121	.0133	.0143	.0397	.0494	.0517	.0519
Γ_q^*	.90	.36	.32	−.019	−.028	.004	.068	.26
Γ_n^*	.0021	.0013	.0010	.0001	−.0003	−.0006	−.0013	−.0033
Γ_δ^*	6.75	2.39	1.88	−.63	−.74	−.55	−.19	1.08

TABLE 1. AIRCRAFT PARAMETERS

(F-4 Aircraft. Units in radians, feet, seconds.)

μ	40	50	60	70	80	90
$H_{\dot{q}}$	.044	.022	.008	−.003	−.011	−.017
H_q	.23	.15	.10	.06	.04	.02
$H_n \times 10^3$	.25	0	−.17	−.29	−.38	−.44
G_0, FC 1	−2.3	−1.7	−1.2	−.90	−.66	−.47
G_0, FC 4	−.77	−.42	−.18	−.01	.11	.21
G_0, FC 5	.30	.44	.53	.60	.65	.69
G_0, FC 8	.45	.56	.63	.68	.72	.75
Roots FC 1	−.64	−.63	−.62	−.62	−.62	−.62
	−5.5	−7.4 + j2.4	−6.0 + j4.0	−4.9 + j4.5	−4.1 + j4.7	−3.5 + j4.8
	−13.8	−7.4 − j2.4	−6.0 − j4.0	−4.9 − j4.5	−4.1 − j4.7	−3.5 − j4.8
Roots FC 4	−3.3 + j5.6	−3.4 + j5.7	−3.4 + j6.0	−3.3 + j6.2	−3.2 + j6.4	−3.1 + j6.5
	−3.3 − j5.6	−3.4 − j5.7	−3.4 − j6.0	−3.3 − j6.2	−3.2 − j6.4	−3.1 − j6.5
	−18.7	−14.1	−11.1	−9.1	−7.7	−6.7
Roots FC 5	−9.7 + j4.6	−10.2 + j9.8	−9.0 + j11.5	−8.1 + j12.3	−7.3 + j12.8	−6.8 + j13.0
	−9.7 − j4.6	−10.2 − j9.8	−9.0 − j11.5	−8.1 − j12.3	−7.3 − j12.8	−6.8 − j13.0
	−11.6	−5.4	−4.2	−3.6	−3.1	−2.9
Root FC 8	−2.1	−1.8	−1.6	−1.4	−1.3	−1.2
	−5.8	−9.7 + j3.5	−7.7 + j6.3	−6.3 + j7.4	−5.3 + j9.9	−4.4 + j8.1
	−19.5	−9.7 − j3.5	−7.7 − j6.3	−6.3 − j7.4	−5.3 − j9.9	−4.4 − j8.1

TABLE 2. ROOT AND GAIN VARIATIONS WITH μ.

APPENDIX

Transfer Functions

Assume the ideal gains (13) obtain. Then the open-loop transfer function, breaking the loop before the actuator, has the numerator

$$N_0 = \left[\beta_\delta H_{\dot q}\left(1 - \frac{\bar\beta_n Z_\delta}{\bar\beta_\delta}\right) - \frac{Z_\delta}{\bar\beta_\delta}\left(\beta_\delta H_n + \beta_n - \bar\beta_n\right)\right] S^2$$

$$+ \left[-\frac{Z_\alpha}{U_0}\beta_\delta H_{\dot q} + \left(1 - \frac{\bar\beta_n Z_\delta}{\bar\beta_\delta}\right)\left(\beta_\delta H_q + \beta_q - \bar\beta_q\right) + \frac{Z_\delta \bar\beta_q}{\bar\beta_\delta}\left(\beta_\delta H_n + \beta_n - \bar\beta_n\right)\right] S$$

$$- \frac{Z_\alpha}{U_0}\left[\left(\beta_\delta H_q + \beta_q - \bar\beta_q\right) + U_0\left(\beta_\delta H_n + \beta_n - \bar\beta_n\right)\right]$$

with denominator of the aircraft and the actuator as

$$\Delta_0 = (T_a S + 1)\left[S^2 - \left(\bar\beta_q + \frac{Z_\alpha}{U_0}\right) S + Z_\alpha\left(\bar\beta_n + \frac{\bar\beta_q}{U_0}\right)\right];$$

the open-loop steady-state gain is

$$G_0 = -\frac{\left(\beta_\delta H_q + \beta_q - \bar\beta_q\right) + U_0\left(\beta_\delta H_n + \beta_n - \bar\beta_n\right)}{\bar\beta_q + U_0 \bar\beta_n}$$

Closing the loop produces the denominator

$$\Delta_c = T_a S^3 + \left[1 - T_a\left(\bar\beta_q + \frac{Z_\alpha}{U_0}\right) - \beta_\delta\left(1 - \frac{\bar\beta_n Z_\delta}{\bar\beta_\delta}\right) H_{\dot q} + \frac{Z_\delta}{\bar\beta_\delta}\left(\beta_\delta H_n + \beta_n - \bar\beta_n\right)\right] S^2$$

$$+ \left[-\bar\beta_q - \frac{Z_\alpha}{U_0} + T_a Z_\alpha\left(\bar\beta_n + \frac{\bar\beta_q}{U_0}\right) + \frac{Z_\alpha}{U_0}\beta_\delta H_{\dot q} - \left(1 - \frac{\bar\beta_n Z_\delta}{\bar\beta_\delta}\right)\left(\beta_\delta H_q + \beta_q - \bar\beta_q\right)\right.$$

$$\left. - \bar\beta_q \frac{Z_\delta}{\bar\beta_\delta}\left(\beta_\delta H_n + \beta_n - \bar\beta_n\right)\right] S + \frac{Z_\alpha}{U_0}\left[\beta_\delta H_q + \beta_q + U_0\left(\beta_\delta H_n + \beta_n\right)\right].$$

After a little algebra, the C*-transfer function is calculated to be

$$\frac{C^*}{C_c} = \frac{\mu \beta_\delta}{T_c S + 1} - \frac{\left(E_2 S^2 + E_1 S + E_0\right) S}{\Delta_c}$$

with

$$\begin{cases} E_0 = \dfrac{Z_\alpha}{U_0}\left[\left(\beta_q - \bar\beta_q\right)\mu T_a - U_0\left(T_c + \mu T_a \beta_n\right)\right] \\[2ex] E_1 = \mu T_a\left(\bar\beta_q - \beta_q\right) + \dfrac{Z_\alpha}{U_0}\left(\mu T_a - T_c \ell\right) + T_c \bar\beta_q \dfrac{Z_\delta}{\bar\beta_\delta} \\[2ex] E_2 = T_c \ell - T_c \dfrac{Z_\delta}{\bar\beta_\delta}\left(1 + \ell \bar\beta_n\right) - \mu T_a . \end{cases}$$

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13. ABSTRACT

A sum of normal acceleration and pitch rate appears to be the best variable to use to control aircraft in the longitudinal axis. The C*-Criterion specifies that the time response of this quantity called C* must fall in a prescribed envelope for all speeds and altitudes. It is equivalent to requiring the control system to hold the coefficients of a certain equation, which describes the aircraft's short-period motion, fixed. This is done by using feedbacks with variable gains. The gain-changing mechanism is found using gradient techniques.

The system was shown to be practical by an analog simulation. It was found to be tolerant of instrument noise, elevator hysteresis, and other complications not accounted in the analytical derivations.

The study strongly suggests a modification of the C*-Criterion. It is proposed that the coefficients in the expression of the quantity C* should be representative of the aircraft's parameters rather than have the values currently in use.

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